Classical Correlation

in the following calculations, kT is considered one variable. So when you see kT^2, it means $k^2\,T^2$in physics.

first term

In[2]:= $$Q = \left(\frac{2\,\pi\,\text{kT}}{\text{h w}}\right)^3;$$

In[3]:= $$\frac{1}{h^3\,Q}\,\underset{\text{积分}}{\text{Integrate}}\Big[\underset{\text{指数形式}}{\text{Exp}}\Big[-\frac{1}{\text{kT}}\left(\frac{\text{p1}^2}{2\,m}+\frac{\text{p2}^2}{2\,m}+\frac{\text{p3}^2}{2\,m}+\frac{1}{2}\,m\,w^2\,\text{x1}^2+\frac{1}{2}\,m\,w^2\,\text{x2}^2+\frac{1}{2}\,m\,w^2\,\text{x3}^2\right)\Big]*\text{x1}^2\,\text{x2}^2,$$
$$\{\text{x1},-\infty,\infty\},\{\text{x2},-\infty,\infty\},\{\text{x3},-\infty,\infty\},$$
$$\{\text{p1},-\infty,\infty\},\{\text{p2},-\infty,\infty\},\{\text{p3},-\infty,\infty\}\Big]\,\text{Cos}[w\,t]^2$$

Out[3]= $$\frac{\text{kT}\,\sqrt{\frac{1}{\text{kT}\,m}}\,\text{Cos}[t\,w]^2}{w\left(\frac{m\,w^2}{\text{kT}}\right)^{3/2}}\quad\text{if}\quad\text{Re}\left[\frac{m\,w^2}{\text{kT}}\right]>0$$

In[ ]:= $$\frac{1}{h^3\,Q}$$
$$\Big(\underset{\text{积分}}{\text{Integrate}}\Big[\underset{\text{指数形式}}{\text{Exp}}\Big[-\frac{1}{\text{kT}}\left(\frac{\text{p1}^2}{2\,m}+\frac{\text{p2}^2}{2\,m}+\frac{\text{p3}^2}{2\,m}+\frac{1}{2}\,m\,w^2\,\text{x1}^2+\frac{1}{2}\,m\,w^2\,\text{x2}^2+\frac{1}{2}\,m\,w^2\,\text{x3}^2\right)\Big]*\text{x1}^4,\{\text{x1},-\infty,$$
$$\infty\},\{\text{x2},-\infty,\infty\},\{\text{x3},-\infty,\infty\},\{\text{p1},-\infty,\infty\},\{\text{p2},-\infty,\infty\},\{\text{p3},-\infty,\infty\}\Big]\,\text{Cos}[w\,t]^2+$$
$$\underset{\text{积分}}{\text{Integrate}}\Big[\underset{\text{指数形式}}{\text{Exp}}\Big[-\frac{1}{\text{kT}}\left(\frac{\text{p1}^2}{2\,m}+\frac{\text{p2}^2}{2\,m}+\frac{\text{p3}^2}{2\,m}+\frac{1}{2}\,m\,w^2\,\text{x1}^2+\frac{1}{2}\,m\,w^2\,\text{x2}^2+\frac{1}{2}\,m\,w^2\,\text{x3}^2\right)\Big]*\frac{\text{x1}^2\,\text{p1}^2}{m^2\,w^2},$$
$$\{\text{x1},-\infty,\infty\},\{\text{x2},-\infty,\infty\},\{\text{x3},-\infty,\infty\},$$
$$\{\text{p1},-\infty,\infty\},\{\text{p2},-\infty,\infty\},\{\text{p3},-\infty,\infty\}\Big]\,\text{Sin}[w\,t]^2\Big)$$

Out[ ]= $$\frac{w^3\left(\frac{24\,\text{kT}^3\,\sqrt{\frac{1}{\text{kT}\,m}}\,m\,\pi^3\,\text{Cos}[t\,w]^2}{w^2\left(\frac{m\,w^2}{\text{kT}}\right)^{5/2}}+\frac{8\,\text{kT}^4\,\sqrt{\frac{1}{\text{kT}\,m}}\,\pi^3\,\text{Sin}[t\,w]^2}{w^4\left(\frac{m\,w^2}{\text{kT}}\right)^{3/2}}\right)}{8\,\text{kT}^3\,\pi^3}\quad\text{if}\quad\text{Re}\left[\frac{m\,w^2}{\text{kT}}\right]>0$$

In[ ]:=
$$\frac{kT\sqrt{\frac{1}{kT\,m}}\,\mathrm{Cos}[t\,w]^2}{w\left(\frac{m\,w^2}{kT}\right)^{3/2}} * 6 + \frac{w^3\left(\frac{24\,kT^3\sqrt{\frac{1}{kT\,m}}\,m\,\pi^3\,\mathrm{Cos}[t\,w]^2}{w^2\left(\frac{m\,w^2}{kT}\right)^{5/2}} + \frac{8\,kT^4\sqrt{\frac{1}{kT\,m}}\,\pi^3\,\mathrm{Sin}[t\,w]^2}{w^4\left(\frac{m\,w^2}{kT}\right)^{3/2}}\right)}{8\,kT^3\,\pi^3} * 3\ //\ \mathrm{FullSimplify}$$
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Out[ ]=
$$\frac{3\sqrt{\frac{1}{kT\,m}}\,m\,w\,(3+2\,\mathrm{Cos}[2\,t\,w])}{\left(\frac{m\,w^2}{kT}\right)^{5/2}}$$

second term

In[ ]:=
$$\frac{1}{h^3\,Q}\,\mathrm{Integrate}\Big[\mathrm{Exp}\Big[-\frac{1}{kT}\left(\frac{p1^2}{2\,m}+\frac{p2^2}{2\,m}+\frac{p3^2}{2\,m}+\frac{1}{2}\,m\,w^2\,x1^2+\frac{1}{2}\,m\,w^2\,x2^2+\frac{1}{2}\,m\,w^2\,x3^2\right)\Big] * x1^2\left(x2^2\,\mathrm{Cos}[w\,t]^2+\frac{p2^2}{m^2\,w^2}\,\mathrm{Sin}[w\,t]^2\right),\ \{x1,-\infty,\infty\},\ \{x2,-\infty,\infty\},\ \{x3,-\infty,\infty\},\ \{p1,-\infty,\infty\},\ \{p2,-\infty,\infty\},\ \{p3,-\infty,\infty\}\Big]$$
积分 指数形式

Out[ ]=
$$\frac{kT\sqrt{\frac{1}{kT\,m}}}{w\left(\frac{m\,w^2}{kT}\right)^{3/2}}\quad \text{if}\quad \mathrm{Re}\left[\frac{m\,w^2}{kT}\right]>0$$

In[ ]:=
$$\frac{1}{h^3\,Q}\,\mathrm{Integrate}\Big[\mathrm{Exp}\Big[-\frac{1}{kT}\left(\frac{p1^2}{2\,m}+\frac{p2^2}{2\,m}+\frac{p3^2}{2\,m}+\frac{1}{2}\,m\,w^2\,x1^2+\frac{1}{2}\,m\,w^2\,x2^2+\frac{1}{2}\,m\,w^2\,x3^2\right)\Big] * x1^2\left(x1^2\,\mathrm{Cos}[w\,t]^2+\frac{p1^2}{m^2\,w^2}\,\mathrm{Sin}[w\,t]^2\right),\ \{x1,-\infty,\infty\},\ \{x2,-\infty,\infty\},\ \{x3,-\infty,\infty\},\ \{p1,-\infty,\infty\},\ \{p2,-\infty,\infty\},\ \{p3,-\infty,\infty\}\Big]$$
积分 指数形式

Out[ ]=
$$\frac{kT^3\left(\frac{1}{kT\,m}\right)^{3/2}(2+\mathrm{Cos}[2\,t\,w])}{w^3\sqrt{\frac{m\,w^2}{kT}}}\quad \text{if}\quad \mathrm{Re}\left[\frac{m\,w^2}{kT}\right]>0$$

In[ ]:=
$$\frac{kT\sqrt{\frac{1}{kT\,m}}}{w\left(\frac{m\,w^2}{kT}\right)^{3/2}} * 6 + \frac{kT^3\left(\frac{1}{kT\,m}\right)^{3/2}(2+\mathrm{Cos}[2\,t\,w])}{w^3\sqrt{\frac{m\,w^2}{kT}}} * 3\ //\ \mathrm{FullSimplify}$$
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Out[ ]=
$$\frac{3\sqrt{\frac{1}{kT\,m}}\,m\,w\,(4+\mathrm{Cos}[2\,t\,w])}{\left(\frac{m\,w^2}{kT}\right)^{5/2}}$$

combine

In[∘]:= $$\frac{3\sqrt{\frac{1}{kT\,m}}\;m\,w\,(3+2\,\mathrm{Cos}[2\,t\,w])}{\left(\frac{m\,w^2}{kT}\right)^{5/2}}-\frac{1}{3}\,\frac{3\sqrt{\frac{1}{kT\,m}}\;m\,w\,(4+\mathrm{Cos}[2\,t\,w])}{\left(\frac{m\,w^2}{kT}\right)^{5/2}}\;\text{// FullSimplify}$$
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Out[∘]= $$\frac{10\sqrt{\frac{1}{kT\,m}}\;m\,w\,\mathrm{Cos}[t\,w]^2}{\left(\frac{m\,w^2}{kT}\right)^{5/2}}$$

In[∘]:= $$10*\frac{kT^2}{m^2\,w^4}\,\mathrm{Cos}^2[w\,t]$$

after Fourier transform

$$\frac{5\,\pi\,k^2\,T^2}{m^2\,w^4}\;\delta[w-2\,w_0]$$

Quantum Correlation

In[4]:= $$QQ=\left(\frac{e^{-\hbar\,w/2\,kT}}{1-e^{-\hbar\,w/kT}}\right)^3;$$

first term

In[∘]:= $$\mathrm{Sum}\Big[\Big((i+1)\,(j+1)\,e^{2\,I\,w\,t}+i\,j\,e^{-2\,I\,w\,t}+(i+1)\,j+i\,(j+1)\Big)\,\mathrm{Exp}\Big[-\frac{1}{kT}\left(i+j+k+\frac{3}{2}\right)\hbar\,w\Big],\;\{i,0,\infty\},\{j,0,\infty\},\{k,0,\infty\}\Big]$$
求和 指数形式

Out[∘]= $$\frac{e^{-2\,i\,t\,w+\frac{3\,w\,\hbar}{2\,kT}}\left(1+e^{2\,i\,t\,w+\frac{w\,\hbar}{kT}}\right)^2}{\left(-1+e^{\frac{w\,\hbar}{kT}}\right)^5}$$

In[∘]:= $$\mathrm{Sum}\Big[\Big((i+1)\,(i+1)\,e^{2\,I\,w\,t}+i\,i\,e^{-2\,I\,w\,t}+2\,(i+1)\,i+2\,i\,(i+1)\Big)\,\mathrm{Exp}\Big[-\frac{1}{kT}\left(i+j+k+\frac{3}{2}\right)\hbar\,w\Big],\;\{i,0,\infty\},\{j,0,\infty\},\{k,0,\infty\}\Big]$$
求和 指数形式

Out[∘]= $$\frac{e^{-2\,i\,t\,w+\frac{3\,w\,\hbar}{2\,kT}}\left(1+e^{\frac{w\,\hbar}{kT}}+8\,e^{2\,i\,t\,w+\frac{w\,\hbar}{kT}}+e^{4\,i\,t\,w+\frac{w\,\hbar}{kT}}+e^{4\,i\,t\,w+\frac{2\,w\,\hbar}{kT}}\right)}{\left(-1+e^{\frac{w\,\hbar}{kT}}\right)^5}$$

In[∘]:=
$$\frac{e^{-2\,i\,t\,w+\frac{3\,w\,\hbar}{2\,kT}}\left(1+e^{2\,i\,t\,w+\frac{w\,\hbar}{kT}}\right)^2}{\left(-1+e^{\frac{w\,\hbar}{kT}}\right)^5}*6+\frac{e^{-2\,i\,t\,w+\frac{3\,w\,\hbar}{2\,kT}}\left(1+e^{\frac{w\,\hbar}{kT}}+8\,e^{2\,i\,t\,w+\frac{w\,\hbar}{kT}}+e^{4\,i\,t\,w+\frac{w\,\hbar}{kT}}+e^{4\,i\,t\,w+\frac{2\,w\,\hbar}{kT}}\right)}{\left(-1+e^{\frac{w\,\hbar}{kT}}\right)^5}*3\ //\ \text{Simplify}$$
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Out[∘]=
$$\frac{3\,e^{-2\,i\,t\,w+\frac{3\,w\,\hbar}{2\,kT}}\left(3+e^{\frac{w\,\hbar}{kT}}+12\,e^{2\,i\,t\,w+\frac{w\,\hbar}{kT}}+e^{4\,i\,t\,w+\frac{w\,\hbar}{kT}}+3\,e^{4\,i\,t\,w+\frac{2\,w\,\hbar}{kT}}\right)}{\left(-1+e^{\frac{w\,\hbar}{kT}}\right)^5}$$

second term

In[∘]:=
$$\text{Sum}\left[\left((i+1)\,(j+1)+(i+1)\,j+i\,(j+1)+i\,j\right)\,\text{Exp}\left[-\frac{1}{kT}\left(i+j+k+\frac{3}{2}\right)\hbar\,w\right],\ \{i,0,\infty\},\{j,0,\infty\},\{k,0,\infty\}\right]$$
求和 指数形式

Out[∘]=
$$\frac{e^{\frac{3\,w\,\hbar}{2\,kT}}\left(1+e^{\frac{w\,\hbar}{kT}}\right)^2}{\left(-1+e^{\frac{w\,\hbar}{kT}}\right)^5}$$

In[∘]:=
$$\text{Sum}\left[\left((i+1)\,(i+1)+(i+1)\,i+i\,(i+1)+i\,i+i\,(i-1)\,e^{-2\,I\,w\,t}+(i+1)\,(i+2)\,e^{2\,I\,w\,t}\right)\text{Exp}\left[-\frac{1}{kT}\left(i+j+k+\frac{3}{2}\right)\hbar\,w\right],\{i,1,\infty\},\{j,0,\infty\},\{k,0,\infty\}\right]+\text{Sum}\left[\left((0+1)\,(0+1)+(0+1)\,(0+2)\,e^{2\,I\,w\,t}\right)\text{Exp}\left[-\frac{1}{kT}\left(0+j+k+\frac{3}{2}\right)\hbar\,w\right],\{j,0,\infty\},\{k,0,\infty\}\right]$$
求和 指数形式 求和 指数形式

Out[∘]=
$$\frac{e^{\frac{w\,\hbar}{2\,kT}}\left(1+2\,e^{2\,i\,t\,w}\right)}{\left(-1+e^{\frac{w\,\hbar}{kT}}\right)^2}+\frac{1}{\left(-1+e^{\frac{w\,\hbar}{kT}}\right)^5}$$
$$e^{-2\,i\,t\,w+\frac{w\,\hbar}{2\,kT}}\left(e^{2\,i\,t\,w}+2\,e^{4\,i\,t\,w}+2\,e^{\frac{w\,\hbar}{kT}}-2\,e^{2\,i\,t\,w+\frac{w\,\hbar}{kT}}-6\,e^{4\,i\,t\,w+\frac{w\,\hbar}{kT}}+9\,e^{2\,i\,t\,w+\frac{2\,w\,\hbar}{kT}}+6\,e^{4\,i\,t\,w+\frac{2\,w\,\hbar}{kT}}\right)$$

In[ ]:=
$$\frac{e^{\frac{3 w \hbar}{2 kT}}\left(1+e^{\frac{w\hbar}{kT}}\right)^2}{\left(-1+e^{\frac{w\hbar}{kT}}\right)^5} * 6 + \left(\frac{e^{\frac{w\hbar}{2 kT}}\left(1+2 e^{2 i t w}\right)}{\left(-1+e^{\frac{w\hbar}{kT}}\right)^2} + \frac{1}{\left(-1+e^{\frac{w\hbar}{kT}}\right)^5} e^{-2 i t w+\frac{w\hbar}{2 kT}}\left(e^{2 i t w}+2 e^{4 i t w}+2 e^{\frac{w\hbar}{kT}}-2 e^{2 i t w+\frac{w\hbar}{kT}}-6 e^{4 i t w+\frac{w\hbar}{kT}}+9 e^{2 i t w+\frac{2 w\hbar}{kT}}+6 e^{4 i t w+\frac{2 w\hbar}{kT}}\right)\right) * 3 \text{ // Simplify}$$

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Out[ ]=
$$\frac{1}{\left(-1+e^{\frac{w\hbar}{kT}}\right)^5} e^{-2 i t w+\frac{3 w\hbar}{2 kT}}\left(6+9 e^{2 i t w}+30 e^{2 i t w+\frac{w\hbar}{kT}}+9 e^{2 i t w+\frac{2 w\hbar}{kT}}+6 e^{4 i t w+\frac{2 w\hbar}{kT}}\right)$$

combine

In[ ]:=
$$\left(\frac{1}{\left(-1+e^{\frac{w\hbar}{kT}}\right)^5} 3 e^{-2 i t w+\frac{3 w\hbar}{2 kT}}\left(3+e^{\frac{w\hbar}{kT}}+12 e^{2 i t w+\frac{w\hbar}{kT}}+e^{4 i t w+\frac{w\hbar}{kT}}+3 e^{4 i t w+\frac{2 w\hbar}{kT}}\right) - \frac{1}{3}\frac{1}{\left(-1+e^{\frac{w\hbar}{kT}}\right)^5} e^{-2 i t w+\frac{3 w\hbar}{2 kT}}\left(6+9 e^{2 i t w}+30 e^{2 i t w+\frac{w\hbar}{kT}}+9 e^{2 i t w+\frac{2 w\hbar}{kT}}+6 e^{4 i t w+\frac{2 w\hbar}{kT}}\right)\right) * \left(\frac{\hbar}{2 m w}\right)^2 / QQ \text{ // Simplify}$$

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Out[ ]=
$$\left(e^{-\frac{1}{2} w\left(4 i t+\frac{3\hbar}{kT}-3 kT \hbar\right)}\left(7-3 e^{2 i t w}+3 e^{\frac{w\hbar}{kT}}+26 e^{2 i t w+\frac{w\hbar}{kT}}+3 e^{4 i t w+\frac{w\hbar}{kT}}-3 e^{2 i t w+\frac{2 w\hbar}{kT}}+7 e^{4 i t w+\frac{2 w\hbar}{kT}}\right)\hbar^2\right) \Big/ \left(4\left(-1+e^{\frac{w\hbar}{kT}}\right)^2 m^2 w^2\right)$$

In[ ]:=
$$\left(e^{-\frac{1}{2} w\left(4 i t+\frac{3\hbar}{kT}-3 kT \hbar\right)}\left(7-3 e^{2 i t w}+3 e^{\frac{w\hbar}{kT}}+26 e^{2 i t w+\frac{w\hbar}{kT}}+3 e^{4 i t w+\frac{w\hbar}{kT}}-3 e^{2 i t w+\frac{2 w\hbar}{kT}}+7 e^{4 i t w+\frac{2 w\hbar}{kT}}\right)\hbar^2\right) \Big/ \left(4\left(-1+e^{\frac{w\hbar}{kT}}\right)^2 m^2 w^2\right) \text{ // FullSimplify}$$

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Out[ ]=
$$\left(e^{\frac{(-1+3 kT^2) w\hbar}{2 kT}}\hbar^2\left(13+3 \text{Cos}[2 t w]+7 \text{Cos}\left[2 t w-\frac{i w\hbar}{kT}\right]-3 \text{Cosh}\left[\frac{w\hbar}{kT}\right]\right)\right) \Big/ \left(2\left(-1+e^{\frac{w\hbar}{kT}}\right)^2 m^2 w^2\right)$$

Then we have to drop all of the real numbers as they only give delta peak at the original point if we do Fourier transform.

Prefactor

In[ ]:=

$$\frac{\frac{e^{\frac{(-1+3\,kT^2)\,w\,\hbar}{2\,kT}}\,\hbar^2}{2\left(-1+e^{\frac{w\,\hbar}{kT}}\right)^2 m^2\,w^2}\left(3+7\,e^{w\,\hbar/kT}\right)}{\frac{5\,kT^2}{m^2\,w^4}}\text{ // FullSimplify}$$

完全简化

Out[ ]=

$$\frac{e^{\frac{(-1+3\,kT^2)\,w\,\hbar}{2\,kT}}\left(3+7\,e^{\frac{w\,\hbar}{kT}}\right)w^2\,\hbar^2}{10\left(-1+e^{\frac{w\,\hbar}{kT}}\right)^2 kT^2}$$